Lecture 4 Part 2 Sampling and Aliasing
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Cellular Generations: 5G, 6G, etc

- New “G” every 10 years (1990 2G, 2000 3G, 2010 4G, 2020 5G, etc.)
- Each “G” should be 10x faster in peak & average bit rates to the user equipment (phone)
- How to get there?
  - Increase the transmission bandwidth
  - Increase the number of antennas at the basestation and user equipment (phone)

Review Of Last Lecture

- Helicopter demo
  - Helicopter lifts off from a helipad on the coast with blades rotating at a certain rate (e.g. 8 revolutions/s)
  - Video camera is taking a video using a fixed frame rate
  - Helicopter blades appear to be still during flight because frame rate causes blades to alias and appear still.
  - Blade frequency (revolutions/s) must be an integer multiple of the frame rate for the blades to appear still in the video
    - Consider taking a picture and then the blade makes one full rotation at the time of the second picture. It appears that the blade hasn’t moved.
    - Consider taking a picture and then the blade makes two full rotations at the time of the second picture. It appears that the blade hasn’t moved….
  - https://www.youtube.com/watch?v=yr3ngmRuGUc

Checkered Ball Demo

- Checkerboard textures on the surfaces of the spheres show places of jagged lines where the black and white squares meet
- Picture left of the vertical line is not filtered, where the picture to the right of the vertical line is filtered by a lowpass filter to smooth the jagged lines
- Divide between black and white squares are sharper – reduces spatial aliasing
- https://www.shadertoy.com/view/XlcSz2

Sampling

- Use lowpass filter to remove frequencies at and above ½ f_s which will alias after sampling by sampling rate f_s
- Model sampling as multiplication by impulse train as in Homework Problem 0.3
- FT of impulse train whose impulses are separated by the sampling time T_s is an impulse train whose impulses are separated by the sampling rate (f_s in Hz or 2pi*f_s in rad/s for continuous time or angular frequency, respectively)
- Resulting FT of sampled signal will include replicas of FT of signal at offsets that are integer multiples of f_s
- Lowpass filter to remove replicas

Sampling Theorem

- Frequencies above ½ f_s cannot be reconstructed
- Unrealistic to assume signal x(t) has no frequency content above ½ f_s, so use lowpass filter to attenuate frequencies above ½ f_s
- Remark: transition band of LPF is ~10% of passband bandwidth in positive frequencies
- What happens to f = ½ f_s? It's the maximum DT frequency
    - Aliasing now depends on the phase of the sinusoidal signal

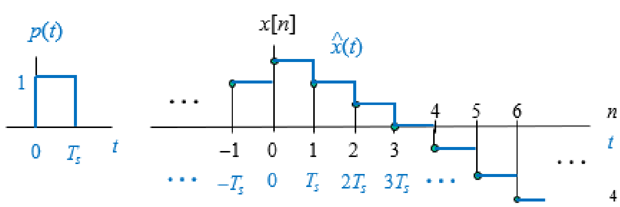
      $x(t) = \cos(2\,\pi\, f_{\max}\, t)$

      $$x[n] = \cos\left(2\,\pi\,\frac{f_{\max}}{f_s}\,n\right) = \cos(\pi\,n) = (-1)^n$$

      $y(t) = \sin(2\,\pi\, f_{\max}\, t)$ gives $y[n] = \sin(\pi\, n) = 0$
    - x(t) and y(t) are separated by a phase shift of pi/2 which leads to x[n] and y[n] separated by a phase shift of pi
    - x[n] is max DT frequency pi, whereas y[n] aliases to 0

Reconstruction

- Want to reconstruct CT signal from DT signal
- Sample and hold
    - Hold amplitude until next sample arrives
    - Equivalent to convolving x[n] with rectangular pulse p(t)
    - Frequency response of rectangular pulse is infinite two-sided sinc

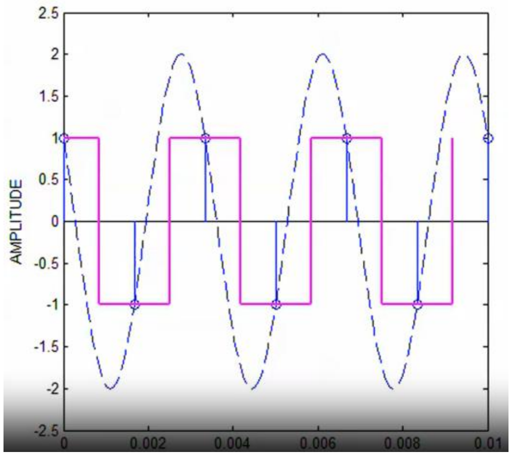


    - Not great reconstruction
    - Effectively adding scaled and shifted rectangular pulses
    - Introduces lots of high frequency content due to the frequency response of p(t) being a two-sided infinite-duration sinc pulse
    - Can apply a LPF to remove high frequency components => remove sharp edges

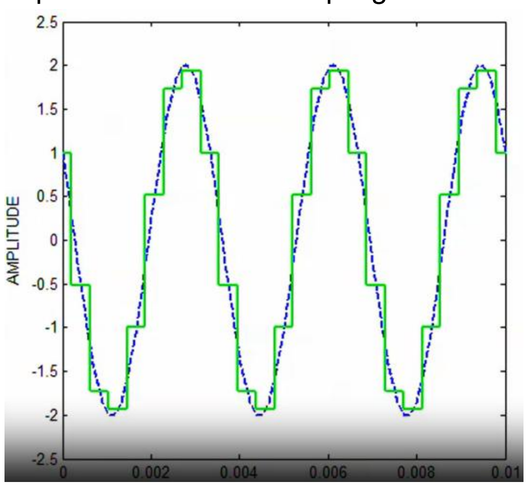
Demos

- All reconstruction filters are lowpass FIR filters with different impulse responses– rectangular pulse, triangular pulse and truncated sinc pulse
- Rectangular pulse/zero order hold
    - Sampling rate is greater than 2 f_0 where f_0 is the cosine frequency
        - Reconstruction using a rectangular pulse looks like square wave, but it were to observe the reconstruction for a longer period of time, the "square" wave would vary in minimum and maximum amplitude
        - Captures fundamental frequency => zero crossings. The distance between two zero crossings is half of the cosine period.

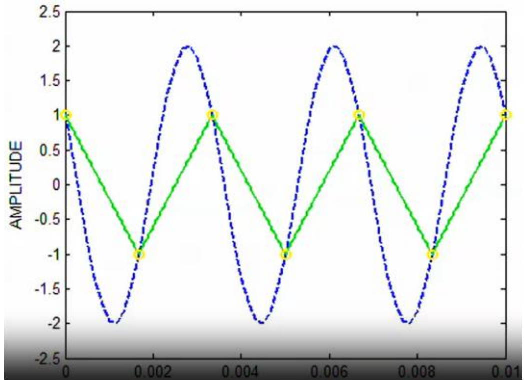
- The location of the zero crossings of the reconstructed signal does not match original CT signal
  - When not sampling exactly at peaks/valleys



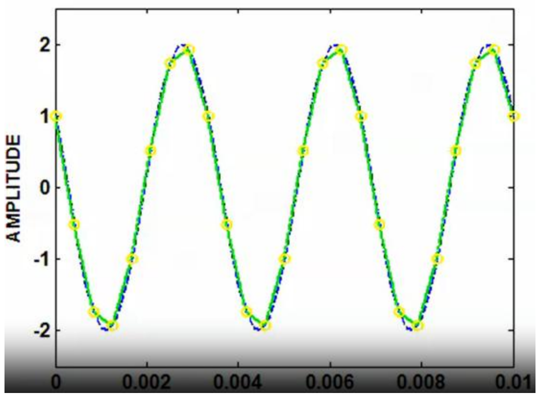
- As the sampling rate increases, the reconstructed signal has
  - peaks/valleys closer in value to the original signal
  - locations of the zero crossings closer to those in the original signal
  - run-time implementation complexity increasing linearly with sampling rate
  - improved signal quality (at some point, signal quality will show negligible improvement as the sampling rate increases: “diminishing returns”)



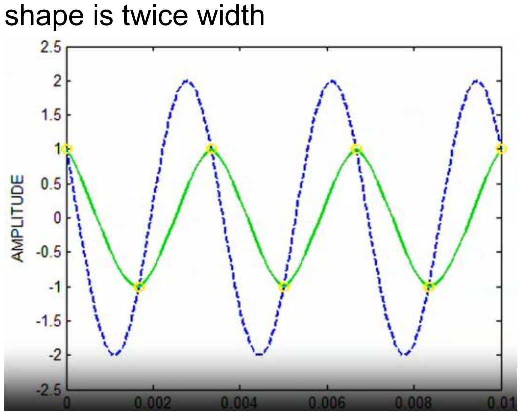
- Linear Interpolation
  - Pulse shape is triangle with width 2*T_s
  - The reconstructed signal appears as lines drawn between sample values like one would do in a “connect the dots” puzzle
  - Sampling rate is greater than 2 f_0
    - Captures correct number of zero crossings
    - Does not capture peaks/valleys and locations of zero crossings



- Sampling rate > 2 f_0
  - Capture peaks/valleys better as sample rate increases
  - Zero crossings are close to CT signal as sample rate increases
  - Doubling sampling rate at least doubles run-time implementation complexity, so eventually get diminishing returns

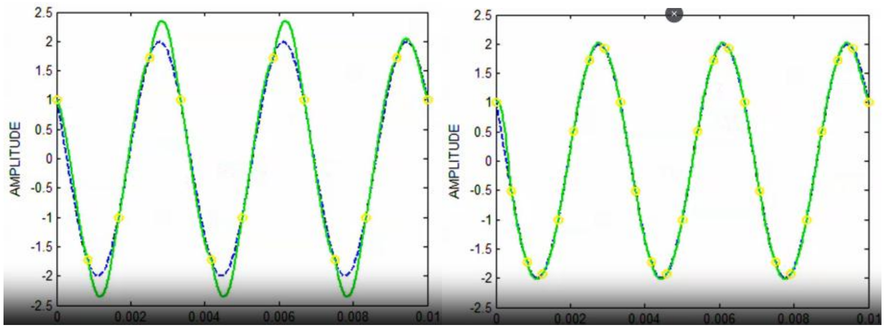


- Truncated Sinc
  - Ideal pulse shape is infinite sinc, cannot implement in practice
  - -2 T_s to 2 T_s
  - Sampling rate > 2 f_0
    - Looks much smoother than similar sample rate for other pulse shapes
    - Double implementation complexity vs. using triangular pulse b/c pulse shape is twice width



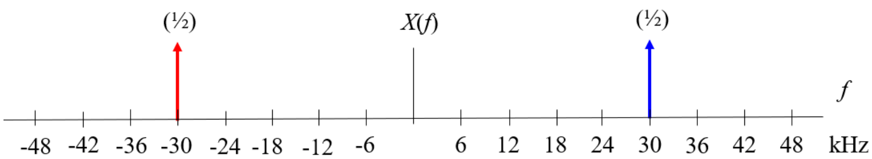
  - Sampling rate > 2f_0

- Better tracking of zero crossings
- For lower sample rates, can overshoot peaks and valleys up to 2x
- Higher sampling rates track peaks/valleys and zero crossings better
- Also eventually see diminishing returns

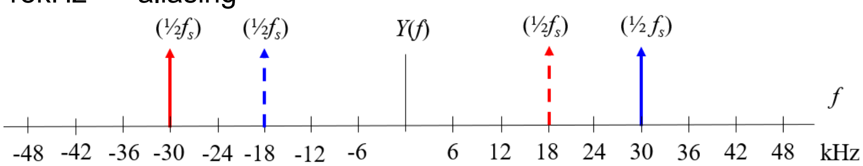


Aliasing

- fs=48kHz, $x(t) = \cos(2\pi f_0 t)$ where $f_0 = 30$ kHz
- X(f):



- Replicas of X(f) centered at multiples of 48kHz will result in frequency content at +/- 18kHz => aliasing



- Reconstruction only uses frequencies $-\frac{1}{2} f_s \leq f \leq \frac{1}{2} f_s$ => signal aliases to 18kHz

Supplemental: After Lecture. Circuit implementation of zero-order-hold:
Disclaimer: Prof. Evans' expertise is in embedded real-time digital systems, not analog circuits.